

# Slot and Microstrip Guiding Structures Using Magnetoplasmons for Nonreciprocal Millimeter-Wave Propagation

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**Abstract**—A full-wave spectral-domain approach for general anisotropy is used to determine the nonreciprocal phase and attenuation properties of slot and microstrip line structures. Dominant mode dispersive behavior is controlled by the semiconductor substrate characteristics, geometric dimensions, and magnetic field bias magnitude and angle in the Voigt configuration. Numerical results are presented to establish the nonreciprocal properties up to 85 GHz.

## I. INTRODUCTION

HERE WE PROVIDE a study of nonreciprocity in slot and microstrip planar structures compatible with millimeter-wave integrated circuit technology. The approach uses a semiconductor to provide a carrier plasma which is then subjected to a static magnetic field  $\vec{B}_0$ . The resulting nondiagonalized form of the semiconductor macroscopic tensor  $\hat{\epsilon}$  and the field displacement effect [1] lead to different propagation constants  $\gamma$  in the forward  $\gamma^+$  and reverse  $\gamma^-$  directions. These unequal  $\gamma$ 's may contain unequal phase propagation constants ( $\beta^+$  and  $\beta^-$ ) and attenuation constants ( $\alpha^+$  and  $\alpha^-$ ). Therefore, both  $\beta$ 's and  $\alpha$ 's are studied to find their absolute values and differences  $\Delta\alpha = \alpha^+ - \alpha^-$  and  $\Delta\beta = \beta^+ - \beta^-$  as functions of  $\phi$ ,  $B_0$ ,  $T$ , and geometric dimensions. Here  $\phi$  and  $T$  are, respectively, the angle of the field in relation to the planar interfaces (inclination angle) and the ambient lattice temperature.

In contrast to the large number of studies done in the past using the Faraday configuration, which requires polarizers to obtain isolator action based upon  $\Delta\alpha = 0$ , here the focus is entirely on the use of the Voigt configuration, where  $\vec{B}_0$  is perpendicular to the direction of propagation. Several recent theoretical investigations [2]–[5] employing the Voigt configuration have examined open, infinite-in-extent layered structures propagating surface waves guided by the dielectric–semiconductor interfaces. The frequency range was from several hundred GHz up to a maximum of between 700 and 1100 GHz. The semiconductor material utilized was GaAs at 77 K and  $10^{15} \text{ cm}^{-3}$  n-type doping,

implying a particular scattering momentum relaxation time  $\tau_p$ . Many branches were found for the structures with  $B_0$  in one orientation, namely parallel to the planar layers ( $\phi = 0^\circ$ );  $B_0$  had been set to  $\approx 3800$  G. An attempt to direct attention toward the millimeter-wave frequency regime was made more recently by another study which treated a semiconductor-loaded waveguide structure [6]. This work considered Si and GaAs semiconductors,  $B_0$  fields up to 10 kG, and several doping densities. Again the Voigt configuration was considered (with  $\phi = 0^\circ$ ). The theoretical analysis used a few waveguide modes. Both theory and experiment had been done at 92 GHz.

## II. MAGNETOPLASMA PERMITTIVITY TENSOR

In this paper the issue of nonreciprocal guided wave propagation in slot and microstrip structures for MMIC's is addressed. The structures are covered and have electric side walls normal to the layers. All conductors are assumed perfect. A full-wave analysis is used which can handle very general linear macroscopic tensors for the layers [7], [8]. Here the anisotropy is restricted to the semiconductor ( $\hat{\epsilon}$ ). A Drude model [9] is employed to describe the individual electron motion in the semiconductor by

$$m^* \frac{d\mathbf{v}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \frac{m^* \mathbf{v}}{\tau_p} \quad (1)$$

where  $q$ ,  $m^*$ , and  $\tau_p$  are, respectively, the electron charge, the effective mass, and the momentum relaxation time. Electron velocity  $\mathbf{v}$  and fields  $\mathbf{E}$  and  $\mathbf{B}$  include dc and RF components. Electron particle current density is

$$\mathbf{J} = qn\mathbf{v} \quad (2)$$

with  $n$  being the electron density and  $q$  the electronic charge.

Particle current density  $\mathbf{J}$  and  $n$  also contain dc and RF components. Decomposition of the variables into dc and RF parts is expressed as [10]

$$\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_{\text{RF}} e^{j\omega t - \gamma z} \quad (3a)$$

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_{\text{RF}} e^{j\omega t - \gamma z} \quad (3b)$$

$$\mathbf{J} = \mathbf{J}_0 + \mathbf{J}_{\text{RF}} e^{j\omega t - \gamma z} \quad (3c)$$

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{v}_{\text{RF}} e^{j\omega t - \gamma z} \quad (3d)$$

$$n = n_0 + n_{\text{RF}} e^{j\omega t - \gamma z} \quad (3e)$$

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where  $\omega$  and  $\gamma$  are, respectively, the electromagnetic radian frequency and the propagation constant in the  $z$  direction. For the case under study, with no electric field, current, or voltage dc biases,  $\mathbf{E}_0 = 0$ ,  $\mathbf{J}_0 = 0$ , and  $\mathbf{v}_0 = 0$ . However, there is an applied dc magnetic field  $\mathbf{B}_0$  given by

$$\begin{aligned}\mathbf{B}_0 &= B_0 [\cos(\phi), \sin(\phi), 0]^T \\ &= B_0 \hat{\mathbf{B}}_0.\end{aligned}\quad (4)$$

Inserting (3) into (1) and (2), extracting out the zeroth- and first-order perturbations, and dropping all higher order nonlinear terms yields

$$0 = q\mathbf{E}_0 + q\mathbf{v}_0 \times \mathbf{B}_0 - \frac{m^* \mathbf{v}_0}{\tau_p} \quad (5a)$$

$$\mathbf{J}_0 = qn_0 \mathbf{v}_0 \quad (5b)$$

and

$$m^* \left( j\omega + \frac{1}{\tau_p} \right) \mathbf{v}_{\text{RF}} = q\mathbf{E}_{\text{RF}} + q\mathbf{v}_{\text{RF}} \times \mathbf{B}_0 \quad (6a)$$

$$\mathbf{J}_{\text{RF}} = q(n_0 \mathbf{v}_{\text{RF}} + n_{\text{RF}} \mathbf{v}_0) = qn_0 \mathbf{v}_{\text{RF}}. \quad (6b)$$

Equations (5) are trivially satisfied by the chosen dc bias conditions, while the RF current  $\mathbf{J}_{\text{RF}}$  in (6b) is simply related by a dc electron proportionality constant to  $\mathbf{v}_{\text{RF}}$ . Eliminating  $\mathbf{v}_{\text{RF}}$  from (6a) using (6b), and defining the plasma  $\omega_p$  and cyclotron  $\omega_c$  radian frequencies as

$$\omega_p^2 = \frac{q^2 n_0}{m^* \epsilon} \quad (7a)$$

$$\omega_c = \frac{qB_0}{m^*} \quad (7b)$$

where  $\epsilon$  is the static semiconductor permittivity, allows a single governing field equation to be written:

$$\left( j\omega + \frac{1}{\tau_p} \right) \mathbf{J}_{\text{RF}} = \epsilon \omega_p^2 \mathbf{E}_{\text{RF}} + \omega_c \mathbf{J}_{\text{RF}} \times \hat{\mathbf{B}}_0. \quad (8)$$

The dc magnetic field unit vector  $\hat{\mathbf{B}}_0$  is defined by (4). Governing equation (8) is recast as

$$\mathbf{E}_{\text{RF}} = \hat{\rho} \mathbf{J}_{\text{RF}} \quad (9)$$

so that the macroscopic resistivity tensor  $\hat{\rho}$  is determined as

$$\hat{\rho} = \begin{bmatrix} j\omega + \tau_p^{-1} & 0 & \omega_c \sin \phi \\ 0 & j\omega + \tau_p^{-1} & -\omega_c \cos \phi \\ -\omega_c \sin \phi & \omega_c \cos \phi & j\omega + \tau_p^{-1} \end{bmatrix} \frac{1}{\epsilon \omega_p^2}. \quad (10)$$

The requirement of a sourceless field problem and a single  $6 \times 6$  constitutive tensor  $\hat{\mathbf{M}}$  relating  $\mathbf{E}_{\text{RF}}$ ,  $\mathbf{D}_{\text{RF}}$ ,  $\mathbf{H}_{\text{RF}}$ , and  $\mathbf{B}_{\text{RF}}$  [7] leads to permeability  $\hat{\mu} = I\mu_0$  ( $\mu_0$  = vacuum value), optical activities  $\hat{\rho}_o = \hat{\rho}_o' = 0$ , and permittivity determined as follows. Maxwell's curl  $\mathbf{H}$  equation is

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J} \quad (11)$$

where  $\mathbf{D}$  is the electric displacement vector field. Here

$$\mathbf{D} = \epsilon \mathbf{E}, \quad (12a)$$

$$\mathbf{H} = \mu_0^{-1} \mathbf{B}. \quad (12b)$$

Placing (9) and (12) into (11), utilizing (3), and retaining the first-order perturbation gives

$$\nabla \times \mathbf{H}_{\text{RF}} = (j\omega \epsilon + \hat{\rho}^{-1}) \mathbf{E}_{\text{RF}} \quad (13)$$

leading to the permittivity (in what comes below, the RF subscripts will be understood and so dropped in the equations):

$$\hat{\epsilon} = \epsilon I + \frac{\hat{\sigma}}{j\omega}, \quad \hat{\sigma} = \hat{\rho}^{-1}. \quad (14)$$

For GaAs,  $\Gamma$  electron valley effective masses are appropriate to use so that

$$\omega_c = 2.63 \times 10^{11} B_0 (10^3 \text{ kG}) \quad (15a)$$

$$\omega_p = 1.93 \times 10^{13} \sqrt{n (10^{17} / \text{cc})}. \quad (15b)$$

Attenuation arises from the finite value of the momentum relaxation time  $\tau_p$  in (10). The off-diagonal  $\hat{\rho}$  elements cause  $\Delta\alpha$  and  $\Delta\beta$  to be nonzero for finite  $B_0$ . Relaxation time  $\tau_p$  can be estimated by a single formula:

$$\tau_p = 3.8048 T^{-0.7361} [20.133 - \log(N_I)] (\text{ps}) \quad (15c)$$

which takes into account ionized impurity scattering ( $N_I$  = ionized density) and phonon scattering. Equation (15c) is based upon experimental and theoretical results for GaAs [11], [12] at lattice temperatures between 77 and 300 K. Numerical calculations done in this paper under different physical parameter conditions are roughly consistent with (15c) for  $\tau_p$  if the semiconductor is envisioned as GaAs. For full ionization,  $n_0 = N_D$  (donor doping) =  $N_I$ .

### III. SLOT ADMITTANCE ANISOTROPIC DYADIC

An admittance-type anisotropic dyadic Green's function, in the spectral domain, is used to relate surface current  $\tilde{\mathbf{J}}$  to tangential electric field  $\tilde{\mathbf{E}}$  at the conductor-slot interface. The one-dimensional finite Fourier transform of an arbitrary two-dimensional spatial function  $f(x, y)$  is

$$\tilde{f}(\alpha_n, y) = \int_{-b/2}^{b/2} f(x, y) e^{-j\alpha_n x} dx \quad (16)$$

where  $y$  = interface and  $\alpha_n = (2n-1)\pi/b$  or  $2n\pi/b$  for even or odd modes, respectively, with  $n$  = any integer. Thus

$$\tilde{J}_x(n) = \tilde{G}'_{11}(\gamma, n) \tilde{E}_x(n) + \tilde{G}'_{12}(\gamma, n) \tilde{E}_z(n) \quad (17a)$$

$$\tilde{J}_z(n) = \tilde{G}'_{21}(\gamma, n) \tilde{E}_x(n) + \tilde{G}'_{22}(\gamma, n) \tilde{E}_z(n). \quad (17b)$$

Dyadic  $\tilde{\mathbf{G}}' = \tilde{\mathbf{G}}^{-1}$ ,  $\tilde{\mathbf{G}}$  being the impedance anisotropic dyadic [8]. Slot electric field components are expanded in terms of complete trigonometric basis function sets satisfying the edge condition. For single slot even and odd modes with respect to  $E_z$ , we use, respectively, the basis functions

$$E_{xm}(x) = \eta_{em}(x) = R^{-1} \sin \left[ (2m-1) \frac{\pi x}{w} \right] \quad (18a)$$

$$E_{zm}(x) = \xi_{em}(x) = R^{-1} \cos \left[ (2m-1) \frac{\pi x}{w} \right] \quad (18b)$$

and

$$E_{xm}(x) = \eta_{om}(x) = R^{-1} \cos \left[ (m-1) \frac{2\pi x}{w} \right] \quad (19a)$$

$$E_{zm}(x) = \xi_{om}(x) = R^{-1} \sin \left[ \frac{2\pi x m}{w} \right] \quad (19b)$$

for  $|x| \leq w/2$  with  $m=1, 2, \dots$ , and  $R = [1 - (2x/w)^2]^{1/2}$ , and the right-hand-sides of (18) and (19) are zero otherwise.

The finite Fourier transforms of (18) and (19) are

$$\begin{aligned} \tilde{E}_{xm}(n) = \tilde{\eta}_{em}(n) = & -\frac{j\pi w}{4} \\ & \cdot \left[ J_0 \left( \alpha_n \frac{w}{2} + (2m-1) \frac{\pi}{2} \right) \right. \\ & \left. - J_0 \left( \alpha_n \frac{w}{2} - (2m-1) \frac{\pi}{2} \right) \right] \quad (20a) \end{aligned}$$

$$\begin{aligned} \tilde{E}_{zm}(n) = \tilde{\xi}_{em}(n) = & \frac{\pi w}{4} \\ & \cdot \left[ J_0 \left( \alpha_n \frac{w}{2} + (2m-1) \frac{\pi}{2} \right) \right. \\ & \left. + J_0 \left( \alpha_n \frac{w}{2} - (2m-1) \frac{\pi}{2} \right) \right] \quad (20b) \end{aligned}$$

for the even mode and

$$\begin{aligned} \tilde{E}_{xm}(n) = \tilde{\eta}_{om}(n) = & \frac{\pi w}{4} \\ & \cdot \left[ J_0 \left( \alpha_n \frac{w}{2} + (m-1)\pi \right) \right. \\ & \left. + J_0 \left( \alpha_n \frac{w}{2} - (m-1)\pi \right) \right] \quad (21a) \end{aligned}$$

$$\begin{aligned} \tilde{E}_{zm}(n) = \tilde{\xi}_{om}(x) = & -\frac{j\pi w}{4} \\ & \cdot \left[ J_0 \left( \alpha_n \frac{w}{2} + m\pi \right) - J_0 \left( \alpha_n \frac{w}{2} - m\pi \right) \right] \quad (21b) \end{aligned}$$

for the odd mode.  $J_0$  is the Bessel function of the first kind and zeroth order. Total slot spectral fields are then

$$\tilde{E}_x(n) = \sum_{m=1}^{n_x} p_m \tilde{E}_{xm}(n) \quad (22a)$$

$$\tilde{E}_z(n) = \sum_{m=1}^{n_z} q_m \tilde{E}_{zm}(n). \quad (22b)$$

Inserting (22) into (17), applying Parseval's theorem, and using a Galerkin approach, a determinantal equation for the propagation constant can be written based on the fact that the slot electric field expansion coefficients are not a trivial null set. Numerical results below are found by setting maximum  $x$  and  $z$  field expansion indices  $n_x$  and  $n_z$  to 1 with maximum  $n \approx 10^2$  (larger  $n_x$  and  $n_z$  values have been tested to ensure solution convergence). Construction of coupled slot electric fields is covered in Ap-

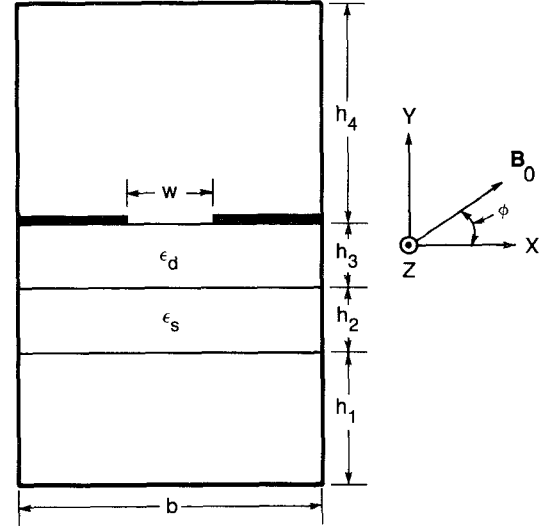


Fig. 1. Cross section of a single suspended slot line structure. Magnetic field bias  $B_0$  is at inclination angle  $\phi$  to the planar surfaces.

pendix I. Derivation of the coupled slot anisotropic determinantal equation is given in Appendix II.

#### IV. NUMERICAL RESULTS

Fig. 1 shows a sketch of a suspended single slot line structure. Electromagnetic propagation is either in the  $z$  direction, out of the paper ("+" or forward direction), or into the paper ("−" or reverse direction). The suspended single slot line rests on two substrates, a dielectric with relative permittivity  $\epsilon_d = 12.5$  and the semiconductor with  $\epsilon_s = \epsilon_d$ . Regions of thickness  $h_1$  and  $h_4$  are air (relative permittivity  $\epsilon_{r1} = \epsilon_{r4} = 1$ , relative permeability  $\mu_r = 1$ ). Slot width  $w = 1.0$  mm, and the other geometric dimensions are  $b = 2.35$  mm,  $h_1 = h_4 = 2.1$  mm, and  $h_2 = 0.25$  mm. Thickness  $h_3$  is varied. Numerical results are first presented for  $\phi = 0^\circ$ , with  $\omega_c = 3.14 \times 10^{12}$  rad/s ( $B_0 = 12$  kG),  $\omega_p = 6.28 \times 10^{12}$  rad/s ( $n_0 = 10^{16}$ /cc), and  $\tau_p = 10^{-13}$  s (room-temperature operation). Only the dominant mode is calculated and discussed here. This mode is odd with respect to  $E_z$  and even with respect to  $E_x$ . In the  $B_0 \rightarrow 0$  limit, the regular dominant slot line mode is obtained.

Fig. 2 shows  $\alpha$  (dB/mm) (with  $\alpha = \alpha^+$  or  $\alpha^-$ ) versus frequency  $f$  (GHz) between 45 and 85 GHz for  $h_3 = 0.25$  mm. At the  $\alpha$  peaks,  $\Delta\alpha/\alpha \approx 15$  percent. The normalized phase propagation constant  $\bar{\beta} = \beta/\beta_0$  with  $\beta = \beta^+$  or  $\beta^-$ ,  $\beta_0$  = free-space value, is shown in Fig. 3(a) and (b) for, respectively,  $h_3 = 0.25$  mm and  $h_3 = 0$  mm. It is evident that  $\Delta\bar{\beta}$  increases with decreasing  $h_3$  by comparing Fig. 3(a) and (b) (and this is particularly apparent in Fig. 5 to follow). Such behavior is physically understandable since  $h_3$  reduction places the magnetoplasma layer in closer proximity to the slot. Here  $\Delta\bar{\beta}/\bar{\beta} \approx 1.3$  percent across the 45 through 80 GHz frequency range displayed ( $h_3 = 0$ ). Figs. 4 and 5 plot, respectively,  $\Delta\alpha$  and  $\Delta\beta$  versus  $f$  between 45 and 75 GHz for  $h_3$  parameterized,  $h_3 = 0, 0.01, 0.1$ , and  $0.25$  mm. Attenuation difference  $\Delta\alpha$  increases with  $f$ , but is insensitive to  $h_3$  variation (Fig. 4).

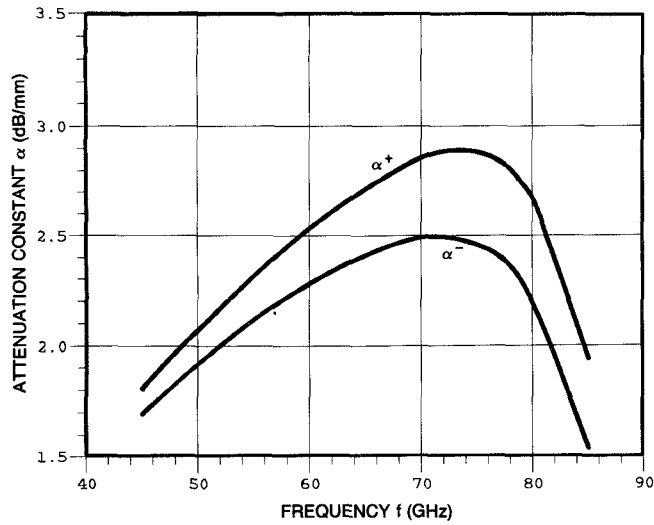


Fig. 2. Dispersion behavior of  $\alpha^+$  and  $\alpha^-$  for the odd mode of single suspended slot line.  $B_0 = 12$  kG,  $\phi = 0^\circ$ ,  $n_0 = 10^{16}/\text{cc}$ ,  $\tau_p = 0.1$  ps,  $w = 1.0$  mm,  $h_1 = h_4 = 2.1$  mm,  $h_2 = h_3 = 0.25$  mm,  $\epsilon_s = \epsilon_d = 12.5$ .

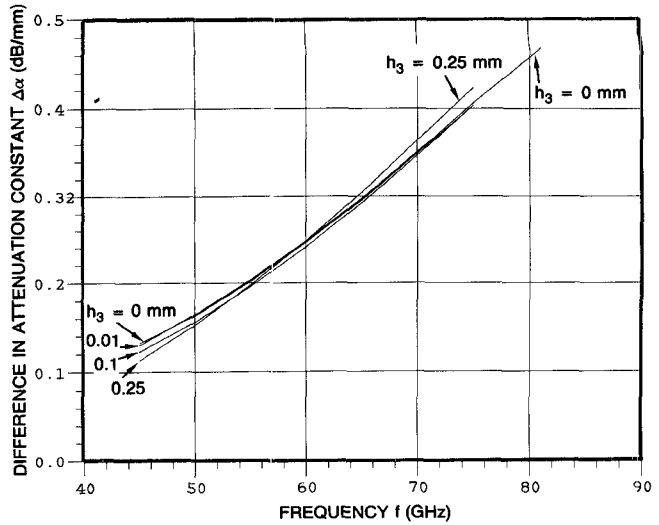
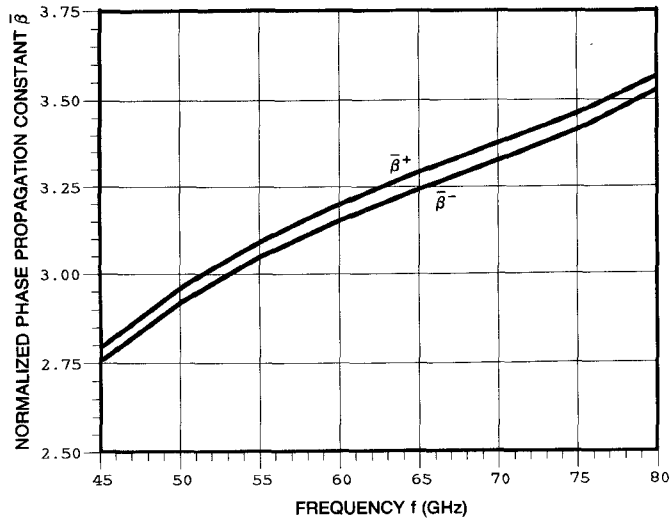
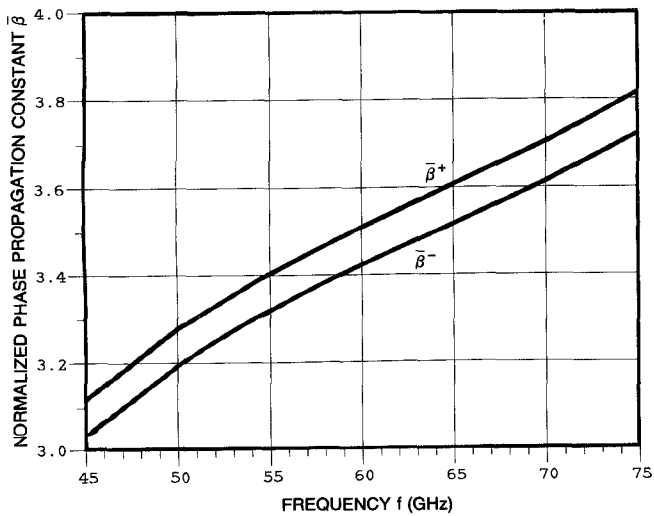


Fig. 4. Dispersion behavior of  $\Delta\alpha$ , with parameterized  $h_3 = 0, 0.01, 0.1$ , and  $0.25$  mm.



(a)



(b)

Fig. 3. (a) Dispersion behavior of  $\bar{\beta}^+$  and  $\bar{\beta}^-$ . Parameters are the same as in Fig. 2. (b) Dispersion behavior of  $\bar{\beta}^+$  and  $\bar{\beta}^-$ . Parameters are as in Fig. 3(a) except  $h_3 = 0$  mm.

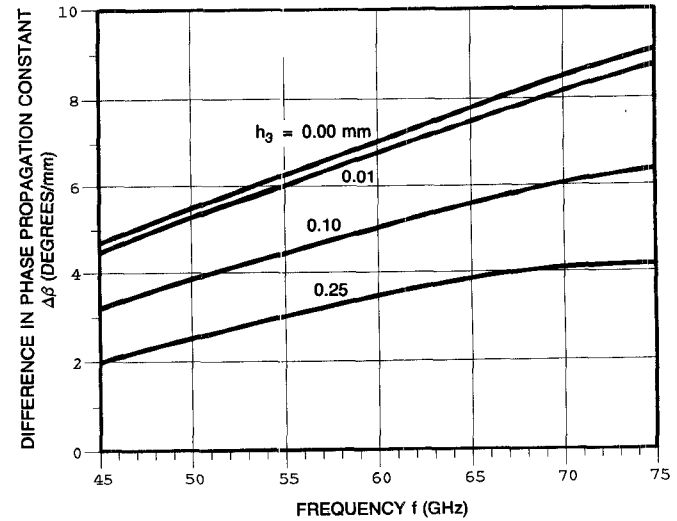
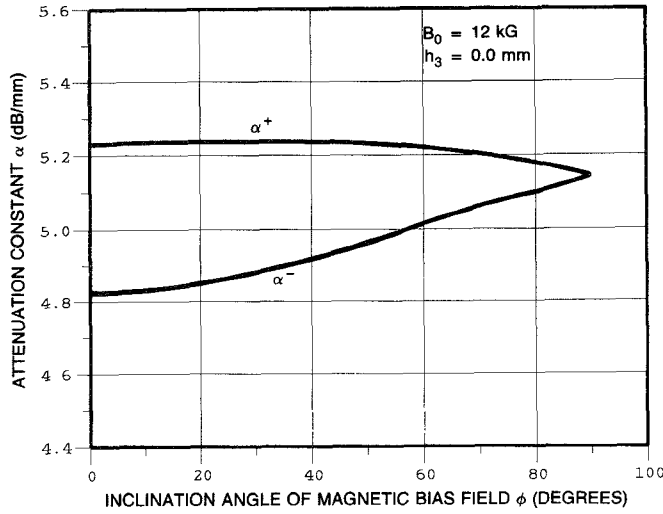
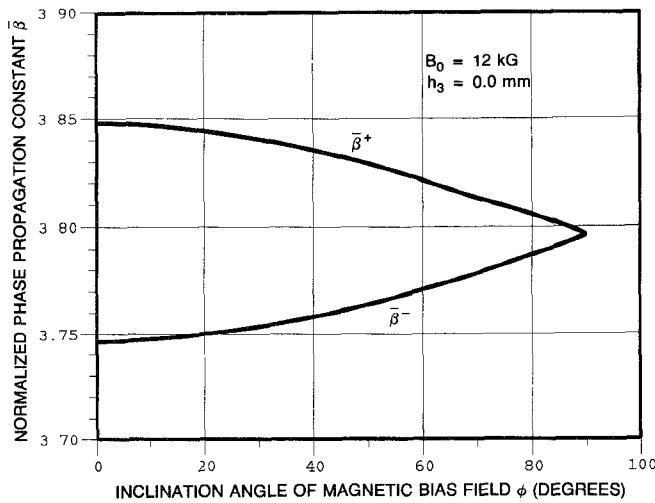
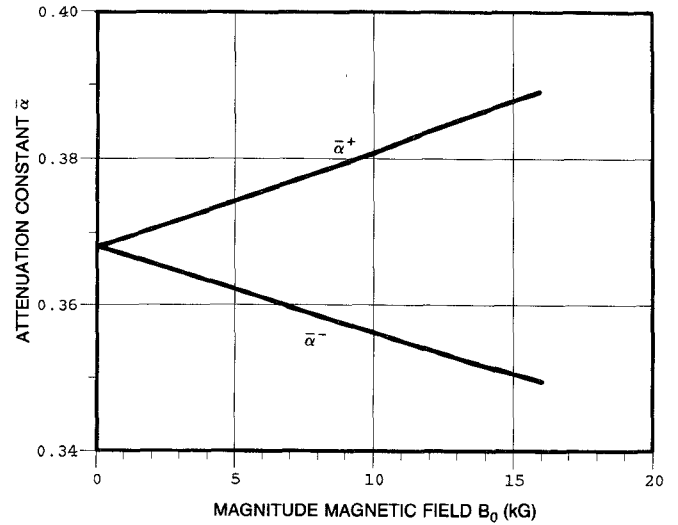


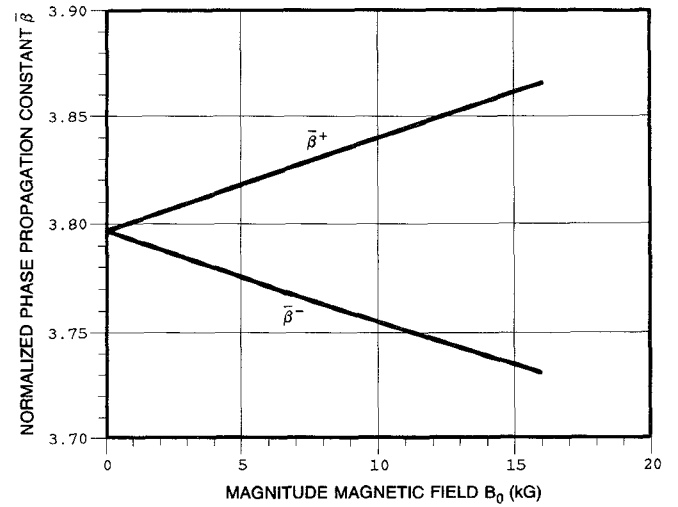
Fig. 5. Dispersion behavior of  $\Delta\beta$ , with  $h_3$  parameterized.

For  $f = 75$  GHz, Figs. 6 and 7 show  $\alpha$  and  $\bar{\beta}$  under varying  $\phi$ . Notice the monotonic decrease of  $\Delta\alpha$  and  $\Delta\bar{\beta}$  with increasing  $\phi$ , and  $\alpha^+ > \alpha^-$  and  $\beta^+ > \beta^-$ . For the choice of parameters selected,  $\Delta\alpha$ ,  $\Delta\bar{\beta}$ ,  $\alpha^-$ ,  $\bar{\beta}^+$  and  $\bar{\beta}^-$  are all roughly constant up to  $15^\circ$  inclination angle. The attenuation constant  $\alpha^+$  hardly varies (compared to  $\alpha^-$ ) up to  $\phi = 60^\circ$ . As  $\phi \rightarrow 90^\circ$ , both  $\Delta\alpha$  and  $\Delta\beta$  approach zero. An explanation for such a limiting characteristic is the creation of cyclotron electron orbits which are perpendicular to the  $B_0$  direction and parallel to the planar interfaces. With  $\phi = 0^\circ$ , Fig. 8(a) and (b) provides  $\bar{\alpha}$  and  $\bar{\beta}$ , respectively, for varying  $B_0$  up to 16 kG.

Four other planar structures in Fig. 9 are now examined for their dominant mode behavior with  $\phi = 0^\circ$ , two slot and two microstrip (see [8] for the forms of the single and coupled microstrips surface currents used in the anisotropic determinantal equation for  $\gamma$  solution). First the even

Fig. 6. Variation of  $\alpha^+$  and  $\alpha^-$  with inclination angle  $\phi$ .Fig. 7. Variation of  $\beta^+$  and  $\beta^-$  with inclination angle  $\phi$ .

(a)



(b)

Fig. 8. (a) Variation of  $\bar{\alpha}^+$  and  $\bar{\alpha}^-$  with magnetic field  $B_0$ .  $h_3 = 0.0$  mm. (b) Variation of  $\bar{\beta}^+$  and  $\bar{\beta}^-$  with magnetic field  $B_0$ .  $h_3 = 0.0$  mm.

mode of the suspended coupled slot structure in Fig. 9(a) is studied for the same geometrical and physical parameters as in Fig. 8 except  $w = w_1 = 0.2$  mm ( $h_3 = 0$  mm and  $f = 75$  GHz). Fig. 10(a) and (b) shows the  $\alpha$  and  $\beta$  variation versus  $B_0$ :  $\Delta\alpha/\alpha \approx 14.2$  percent and  $\Delta\beta/\beta \approx 1.6$  percent at  $B_0 = 10$  kG. Secondly the odd mode of the sandwiched suspended single slot structure in Fig. 9(b) was studied under a varying  $B_0$  field for the same parameters as in Fig. 10 except  $h_2 = h_3 = 0.25$  mm and  $w = 1.0$  mm. One finds that  $\alpha(0 \text{ kG}) = 3.196$  dB/mm,  $\bar{\beta}(0 \text{ kG}) = 3.173$ ,  $\alpha^-(12 \text{ kG}) = 2.965$  dB/mm,  $\bar{\beta}^-(12 \text{ kG}) = 3.155$ , and  $\Delta\alpha/\alpha = 15.2$  percent and  $\Delta\bar{\beta}/\bar{\beta} = 1.1$  percent at 12 kG. Intermediate  $\gamma(B_0)$  values are easily found due to the linearity of the curves and the ordering property  $\alpha^+ > \alpha^-$ ,  $\bar{\beta}^+ > \bar{\beta}^-$ . The ordering property occurs for most of our results and associates greater slowing with higher attenuation. Comparison to the normalized attenuation constant  $\bar{\alpha} = \alpha/\beta_0$  in Fig. 8(a) indicates about a 40 percent reduction in  $\alpha$ , suggesting that the addition of a dielectric cover of this thickness removes much of the field intensity from

the semiconductor. On the other hand, thinning  $h_3$  to  $h_3 = 0.10$  mm increases  $\alpha$  by about 25 percent, an effect which may be attributed to pulling the fields into closer proximity to the slot region. One has  $\alpha(0 \text{ kG}) = 6.77$  dB/mm,  $\alpha^-(14 \text{ kG}) = 6.502$  dB/mm, and  $\Delta\alpha/\alpha = 8.2$  percent. Fig. 11(a) and (b) shows the behavior of  $\alpha$  and  $\bar{\beta}$  as  $h_2$  is varied from about 0.07 mm to 1.2 mm at  $B_0 = 12$  kG with  $h_3 = 0.10$  mm and  $w = 1.0$  mm (Fig. 9(b) structure; other parameters the same as in Fig. 10). Here  $\alpha^{+,-}$  first rises (as expected since  $\alpha \equiv 0$  at  $h_2 = 0$ ), then monotonically decreases—similar behavior is observed for  $\bar{\beta}$ . The precipitous, then gradual decrease of  $\gamma^{+,-}$  with  $h_2$  is related to the reduction of fields located in the slot vicinity. Thirdly the even mode of the suspended single microstrip structure in Fig. 9(c) is analyzed under a varying  $B_0$  field for the same parameters as in Fig. 10 except  $h_3 = 0$  mm and  $w_1 = 0.2$  mm. It is found that  $\gamma$  depends linearly on  $B_0$  and that  $\alpha(0 \text{ kG}) = 4.021$  dB/mm,  $\alpha^-(10 \text{ kG}) = 3.832$  dB/mm, and  $\Delta\alpha/\alpha = 9.8$  percent at 10 kG. Shrinking  $w_1$  to 0.0125 mm causes  $\alpha$  reduction ( $\alpha(0 \text{ kG}) =$

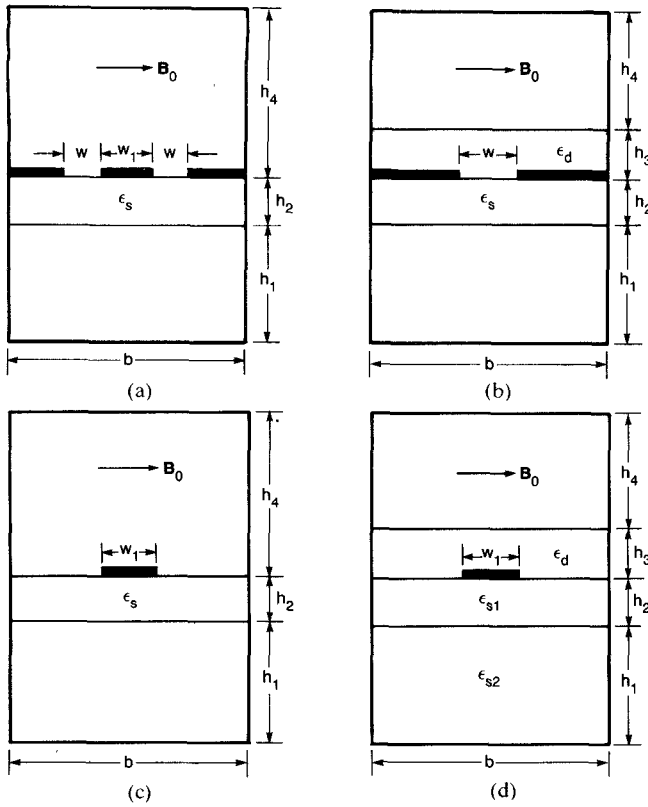


Fig. 9. Cross sections of planar structures possessing a semiconductor(s) under dc magnetic field bias  $B_0$  and inclination angle  $\phi = 0^\circ$ . The gyroelectric effect is studied in (a) suspended coupled slot line, (b) sandwiched suspended single slot line, (c) suspended single microstrip, and (d) covered single microstrip.

2.835 dB/mm,  $\alpha^-$  (12 kG) = 2.631 dB/mm) but  $\Delta\alpha/\alpha = 15.8$  percent at 12 kG.

The last structure examined is the dielectric-covered single microstrip over a doped semiconductor layer of thickness  $h_2 = 0.25$  mm and an undoped substrate of thickness  $h_1 = 2.1$  mm ( $= h_4$ ). It is shown in Fig. 9(d). Here the cover has  $h_3 = 0.1$  mm,  $w_1 = 0.2$  mm, and  $\epsilon_d = \epsilon_{s1} = \epsilon_{s2} = 12.5$ . Fig. 12(a) and (b) gives  $\alpha$  and  $\bar{\beta}$  versus  $B_0$  up to nearly 35 kG at  $f = 50$  GHz with  $\omega_p = 6.28 \times 10^{11}$  rad/s ( $n_0 = 10^{14}$ /cc). The even mode results are parameterized in terms of  $\tau_p = (1, 5, 10) \times 10^{-13}$  s. At low  $B_0$ ,  $\alpha$  increases as  $\tau_p$  increases, an effect not seen beyond 5 kG. As  $\tau_p \rightarrow \infty$ ,  $\alpha \rightarrow 0$  (not shown), as expected for no carrier scattering. The quantity  $\Delta\alpha$  first increases then decreases with rising  $B_0$ . Maximum  $\Delta\alpha \approx 1.4$  dB/mm and occurs for  $3 \leq B_0 \leq 7$  kG with  $\tau_p = 10^{-12}$  s. This corresponds to a  $\Delta\alpha/\alpha \approx 47$  percent at 5 kG. One notices that these curves are extremely nonlinear for  $B_0 < 20$  kG and  $\tau_p \neq 10^{-13}$  s. Highly nonlinear behavior is mimicked in the  $\bar{\beta}$  versus  $B_0$  curves for  $B_0 < 20$  kG and  $\tau_p \neq 10^{-13}$  s (Fig. 12(b)). Beyond 20 kG,  $\Delta\bar{\beta}$  is approximately constant and  $\bar{\beta}$  behaves linearly. Attenuation dependence of  $\alpha$  on  $\tau_p$  at  $B_0 = 20$  kG is presented in Figs. 13(a) and (b) for different  $n_0 = 10^{12}$ ,  $10^{13}$ , and  $10^{14}$ /cc. As  $\tau_p \rightarrow 10^{-14}$  s, both  $\alpha$  and  $\Delta\alpha$  decrease. Also as expected,  $\alpha$  and  $\Delta\alpha$  tend to decrease as  $\tau_p$  gets large (beyond  $\tau_p = 10^{-11}$  s for  $n_0 = 10^{14}$ /cc). However,  $\Delta\alpha/\alpha \approx 130$  percent near  $\tau_p = 6.2 \times 10^{-12}$  s. When  $n_0$  in-

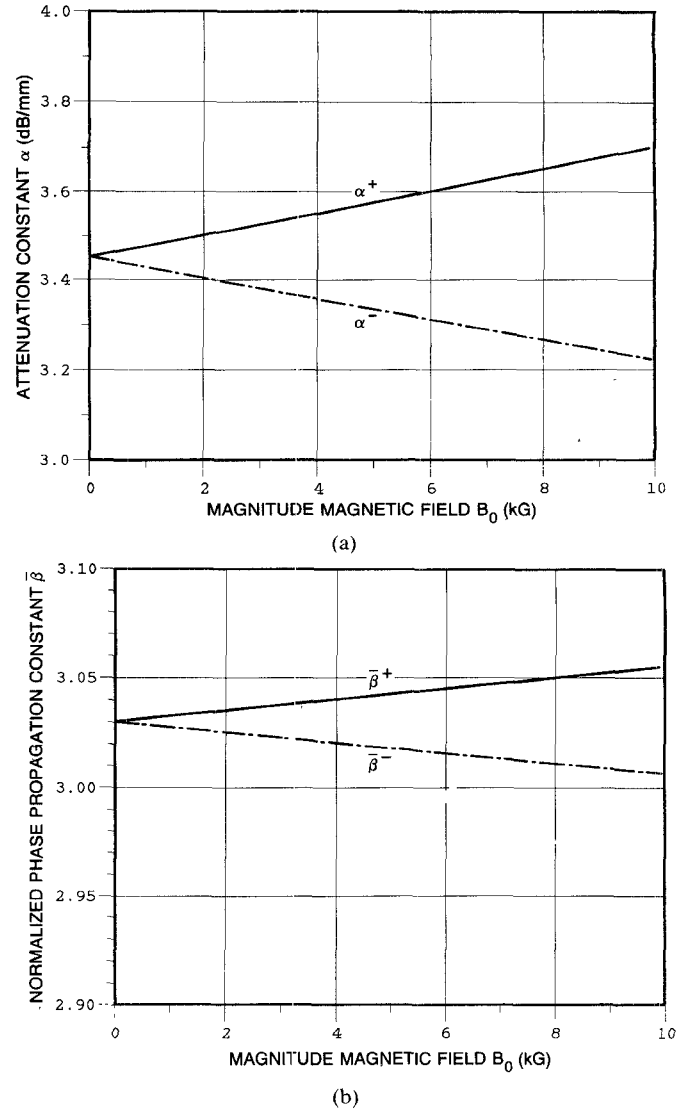
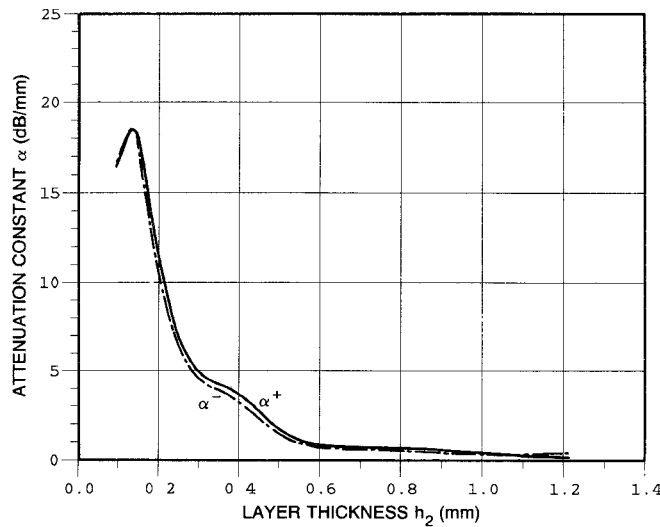


Fig. 10. (a) Variation of  $\alpha^+$  and  $\alpha^-$  with magnetic field  $B_0$  for the even mode of the suspended coupled slot structure (Fig. 9(a)). Same parameters as used in Fig. 2 except  $h_3 = 0$  mm,  $w = w_1 = 0.2$  mm, and  $f = 75$  GHz. (b) Variation of  $\bar{\beta}^+$  and  $\bar{\beta}^-$  with magnetic field  $B_0$  for the Fig. 10(a) case.

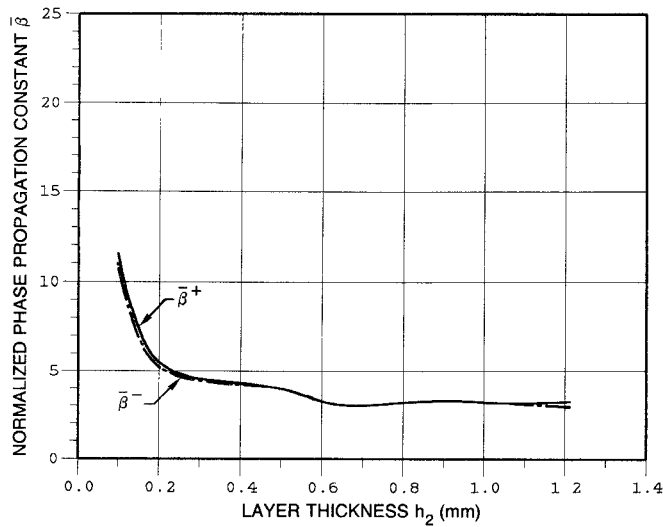
creases, the absolute  $\alpha$  maxima move to the left (or smaller  $\tau_p$ ). Fig. 14 presents the corresponding  $\bar{\beta}$  versus  $\tau_p$  dependence.  $\Delta\bar{\beta}$  increases with increasing  $n_0$ . For  $n_0 = 10^{14}$ /cc,  $\Delta\bar{\beta} \approx$  constant up to  $\tau_p = 2.5 \times 10^{-12}$  s, beyond which it monotonically increases to  $\Delta\bar{\beta}/\bar{\beta} \approx 11$  percent. Fig. 15(a) and (b) provides  $\alpha$  and  $\bar{\beta}$  against  $h_2$  for  $B_0 = 10$  kG,  $\tau_p = 10^{-13}$  s, and  $w_1 = 0.2$  mm with parameterized  $h_3$ .

## V. CONCLUSION

A full-wave matrix spectral-domain approach for complex anisotropy is used to find the propagation constants in the forward and reverse directions for suspended single slot line layered structures. A dc bias magnetic field  $B_0$  is employed to generate permittivity anisotropy in the semiconductor layer. The  $B_0$  field is normal to the propagation direction, and its inclination angle  $\phi$  to the interface is varied. Numerical results between 45 and 85 GHz for a GaAs semiconductor layer are obtained. Differences between forward and reverse attenuation constants (dB/ $\lambda_0$ )



(a)



(b)

Fig. 11. (a) Variation of  $\alpha^+$  and  $\alpha^-$  against  $h_2$  for the even mode of the sandwiched suspended single slot structure (Fig. 9(b)). Same parameters as in Fig. 10 except  $h_3 = 0.10$  mm,  $w = 1.0$  mm, and  $B_0 = 12$  kG. (b) Variation of  $\beta^+$  and  $\beta^-$  against  $h_2$  for the Fig. 11(a) case.

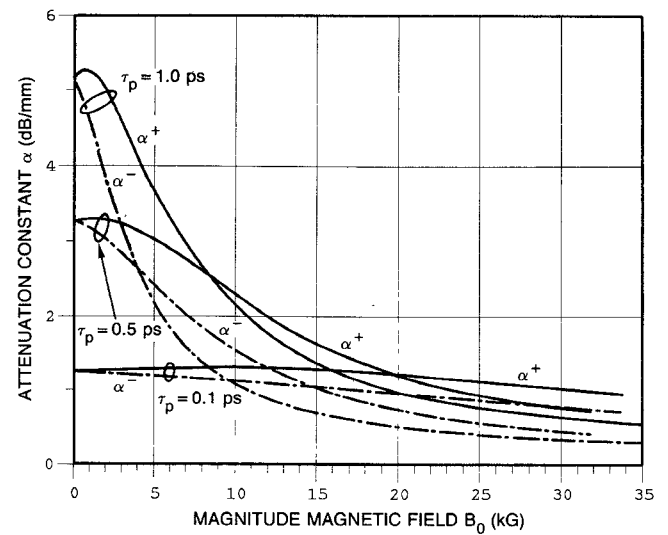
and phase constants (degrees/ $\lambda_0$ ) show increases with increasing frequency. These differences appear to be largest, though, for small  $\phi \leq 20^\circ$ .

Numerical results are also found for other slot line structures (sandwich suspended single slot and suspended coupled slot) and single microstrip structures (covered microstrip and suspended microstrip) for  $\phi = 0^\circ$ . Physical parameters of the doped semiconductor such as  $n_0$  and  $\tau_p$  as well as  $B_0$  and geometric parameters are varied to characterize nonreciprocal effects.

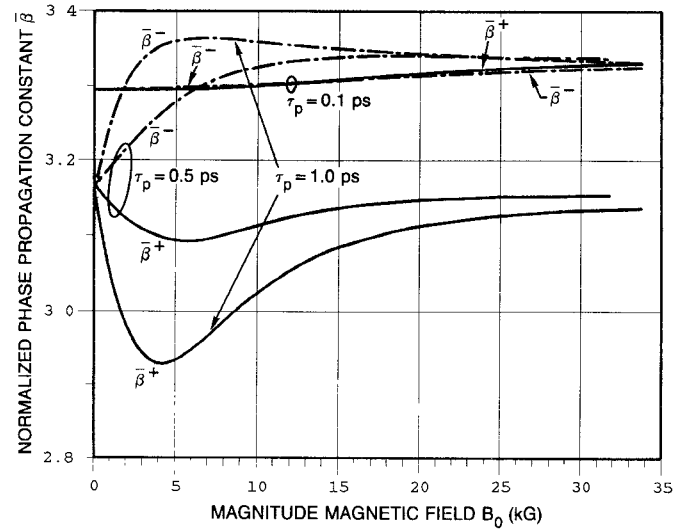
#### APPENDIX I

Coupled slot electric field (only within the slots, zero otherwise) are expressed as

$$E_x(x) = \sum_{m=1}^{n_x} \{a_{m1}\eta_{em}(x+s) + b_{m1}\eta_{om}(x+s)\} + \sum_{m=1}^{n_x} \{a_{m2}\eta_{em}(x-s) + b_{m2}\eta_{om}(x-s)\} \quad (A1)$$



(a)



(b)

Fig. 12. (a) Variation of  $\alpha^+$  and  $\alpha^-$  with magnetic field  $B_0$  for the even mode of the covered single microstrip structure (Fig. 9(d)). Parameters same as in Fig. 11 except that here  $h_2 = 0.25$  mm,  $h_3 = 0.10$  mm,  $\epsilon_d = \epsilon_{s1} = \epsilon_{s2} = 12.5$ ,  $n_0 = 10^{14}$ /cc, and  $f = 50$  GHz. Results are parameterized in  $\tau_p$ . (b) Variation of  $\beta^+$  and  $\beta^-$  with magnetic field  $B_0$  for the Fig. 12(a) case.

$$E_z(x) = \sum_{m=1}^{n_z} \{c_{m1}\xi_{em}(x+s) + d_{m1}\xi_{om}(x+s)\} + \sum_{m=1}^{n_z} \{c_{m2}\xi_{em}(x-s) + d_{m2}\xi_{om}(x-s)\}. \quad (A2)$$

Here the first summed term in (A1) and (A2) accounts for the slot located at  $x = -s$ ,  $s = (w + w_1)/2$ , whereas the second term accounts for the slot located at  $x = +s$ . Basis functions  $\eta_{em}$ ,  $\eta_{om}$ ,  $\xi_{em}$ ,  $\xi_{om}$  are even and odd single slot functions provided in (18) and (19).

For the coupled slot even mode case,

$$E_z(x) = E_z(-x) \quad (A3a)$$

$$E_x(x) = -E_x(-x). \quad (A3b)$$

Placing (A2) into (A3a) yields, for the  $m$ th term,

$$\begin{aligned} & c_{m1}\xi_{em}(x+s) + d_{m1}\xi_{om}(x+s) + c_{m2}\xi_{em}(x-s) \\ & + d_{m2}\xi_{om}(x-s) \\ & = c_{m1}\xi_{em}(-x+s) + d_{m1}\xi_{om}(-x+s) \\ & + c_{m2}\xi_{em}(-x-s) + d_{m2}\xi_{om}(-x-s) \\ & = c_{m1}\xi_{em}(x-s) - d_{m1}\xi_{om}(x-s) \\ & + c_{m2}\xi_{em}(x+s) - d_{m2}\xi_{om}(x+s) \end{aligned}$$

or

$$\begin{aligned} & (c_{m1} - c_{m2})\xi_{em}(x+s) + (d_{m1} + d_{m2})\xi_{om}(x+s) \\ & + (c_{m2} - c_{m1})\xi_{em}(x-s) \\ & + (d_{m2} + d_{m1})\xi_{om}(x-s) = 0 \end{aligned}$$

or, more compactly,

$$(c_{m1} - c_{m2})[\xi_{em}(x+s) - \xi_{em}(x-s)] + (d_{m1} + d_{m2}) \cdot [\xi_{om}(x+s) + \xi_{om}(x-s)] = 0. \quad (\text{A4})$$

Equation (A4) will only hold if

$$c_{m1} = c_{m2} \quad d_{m1} = -d_{m2}. \quad (\text{A5})$$

Inserting (A5) into (A2) gives the spatial  $E_z$  dependence,

$$E_z(x) = \sum_{m=1}^{n_z} \{ c_{m1}[\xi_{em}(x+s) + \xi_{em}(x-s)] + d_{m1} \cdot [\xi_{om}(x+s) - \xi_{om}(x-s)] \}. \quad (\text{A6})$$

Following a similar procedure for  $E_x(x)$  produces

$$a_{m1} = -a_{m2} \quad b_{m1} = b_{m2} \quad (\text{A7})$$

and

$$E_x(x) = \sum_{m=1}^{n_x} \{ a_{m1}[\eta_{em}(x+s) - \eta_{em}(x-s)] + b_{m1} \cdot [\eta_{om}(x+s) + \eta_{om}(x-s)] \}. \quad (\text{A8})$$

By employing the Fourier shifting relationships

$$\tilde{\xi}_{e,om}(x \pm s) = e^{\pm j\alpha_n s} \tilde{\xi}_{e,om}(n) \quad (\text{A9a})$$

$$\tilde{\eta}_{e,om}(x \pm s) = e^{\pm j\alpha_n s} \tilde{\eta}_{e,om}(n) \quad (\text{A9b})$$

where the arguments of the left-hand side variables are present to identify the spatial variables transformed, the final forms of spectral slot electric fields are stated, using (A6) and (A8), as

$$\tilde{E}_x(n) = \sum_{m=1}^{n_x} \{ a'_{m1} \sin(\alpha_n s) \tilde{\eta}_{em} + b'_{m1} \cos(\alpha_n s) \tilde{\eta}_{om} \} \quad (\text{A10a})$$

$$\tilde{E}_z(n) = \sum_{m=1}^{n_z} \{ c'_{m1} \cos(\alpha_n s) \tilde{\xi}_{em} + d'_{m1} \sin(\alpha_n s) \tilde{\xi}_{om} \}. \quad (\text{A10b})$$

Here the primed coefficients are related to the old coeffi-

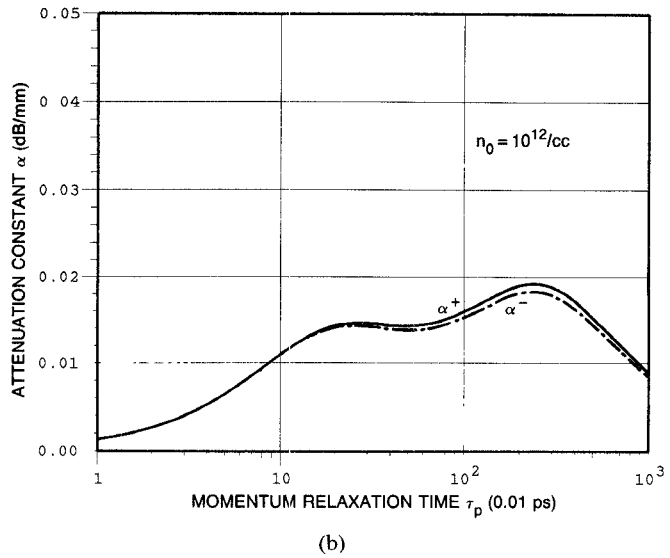
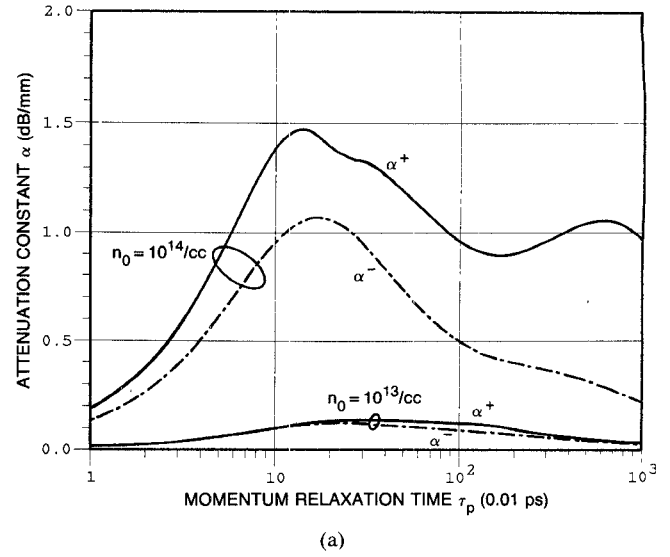


Fig. 13. (a) Variation of  $\alpha^+$  and  $\alpha^-$  against  $\tau_p$  for the Fig. 12(a) case with  $B_0 = 20$  kG. Parameterized curves for  $n_0 = 10^{13}$  and  $10^{14}/\text{cc}$ . (b) Variation of  $\alpha^+$  and  $\alpha^-$  against  $\tau_p$  for the Fig. 13(a) case.  $n_0 = 10^{12}/\text{cc}$ .

cients as follows:

$$\begin{aligned} a'_{m1} &= 2ja_{m1} & b'_{m1} &= 2b_{m1} \\ c'_{m1} &= 2c_{m1} & d'_{m1} &= 2jd_{m1}. \end{aligned} \quad (\text{A11})$$

For the coupled slot odd mode case,

$$E_z(x) = -E_z(-x) \quad E_x(x) = E_x(-x). \quad (\text{A12})$$

Using the same reasoning process as for the even mode case,

$$\tilde{E}_x(n) = \sum_{m=1}^{n_x} \{ a''_{m1} \cos(\alpha_n s) \tilde{\eta}_{em} + b''_{m1} \sin(\alpha_n s) \tilde{\eta}_{om} \} \quad (\text{A13a})$$

$$\tilde{E}_z(n) = \sum_{m=1}^{n_z} \{ c''_{m1} \sin(\alpha_n s) \tilde{\xi}_{em} + d''_{m1} \cos(\alpha_n s) \tilde{\xi}_{om} \} \quad (\text{A13b})$$

where

$$\begin{aligned} a''_{m1} &= 2a_{m1} & b''_{m1} &= 2jb_{m1} \\ c''_{m1} &= 2jc_{m1} & d''_{m1} &= 2d_{m1}. \end{aligned} \quad (\text{A14})$$



## APPENDIX II

The coupled slot anisotropic determinantal equation is derived as follows. First find a Parseval theorem. At the interface of interest, we drop  $y$  in (10) to obtain

$$\Lambda(x) = \frac{1}{b} \sum_{n=-\infty}^{\infty} \tilde{\Lambda}(n) e^{j\alpha_n x} \quad (\text{A15})$$

$$\tilde{\Lambda}(n) = \int_{-b/2}^{b/2} \Lambda(x) e^{-j\alpha_n x} dx \quad (\text{A16})$$

Letting  $\Lambda = g$  or  $f$ ,

$$\begin{aligned} \sum_{n=-\infty}^{\infty} \tilde{f}(n) \tilde{g}(n) &= \sum_{n=-\infty}^{\infty} \tilde{g}(n) \int_{-b/2}^{b/2} f(x) e^{-j\alpha_n x} dx \\ &= - \int_{-b/2}^{b/2} f(-x') \left[ \sum_{n=-\infty}^{\infty} \tilde{g}(n) e^{j\alpha_n x'} \right] dx' \\ &= -b \int_{-b/2}^{b/2} f(-x) g(x) dx. \end{aligned} \quad (\text{A17})$$

If one considers  $g = J_x$  or  $J_z$  as in (17), and  $f$  equal to the  $\xi_{em}$  (or  $\eta_{em}$ ) or  $\xi_{om}$  (or  $\eta_{om}$ ) parts (indicate parts by  $\xi'_{(e,o)m}$  or  $\eta'_{(e,o)m}$ ) of  $E_z$  or  $E_x$  in (A6) or (A8), noting that  $f(-x) = \pm f(x)$  depending on symmetry, the assumption of perfect conductors leading to complementary current and field parts makes the right-hand side of (A17) zero.

Now examine (17) relating coupled slot fields and surface currents by the anisotropic  $\tilde{G}$  dyadic. Multiply (17a) by  $\tilde{\eta}'_{ej}$ ,  $\tilde{\eta}'_{oj}$  and (17b) by  $\tilde{\xi}'_{ej}$ ,  $\tilde{\xi}'_{oj}$ ; then the spectral sum (we can employ  $n=1,2,\dots$  due to symmetry if desired) is

$$\begin{aligned} \sum_{n=-\infty}^{\infty} \tilde{\eta}'_{ej} \tilde{G}'_{11}(\gamma, n) \tilde{E}_x(n) + \sum_{n=-\infty}^{\infty} \tilde{\eta}'_{ej} \tilde{G}'_{12}(\gamma, n) \\ \times \tilde{E}_z(n) = 0, \quad j=1,2,\dots,n_x \end{aligned} \quad (\text{A18a})$$

$$\begin{aligned} \sum_{n=-\infty}^{\infty} \tilde{\eta}'_{oj} \tilde{G}'_{11}(\gamma, n) \tilde{E}_x(n) + \sum_{n=-\infty}^{\infty} \tilde{\eta}'_{oj} \tilde{G}'_{12}(\gamma, n) \\ \times \tilde{E}_z(n) = 0, \quad j=1,2,\dots,n_x \end{aligned} \quad (\text{A18b})$$

$$\begin{aligned} \sum_{n=-\infty}^{\infty} \tilde{\xi}'_{ej} \tilde{G}'_{21}(\gamma, n) \tilde{E}_x(n) + \sum_{n=-\infty}^{\infty} \tilde{\xi}'_{ej} \tilde{G}'_{22}(\gamma, n) \\ \times \tilde{E}_z(n) = 0, \quad j=1,2,\dots,n_z \end{aligned} \quad (\text{A18c})$$

$$\begin{aligned} \sum_{n=-\infty}^{\infty} \tilde{\xi}'_{oj} \tilde{G}'_{21}(\gamma, n) \tilde{E}_x(n) + \sum_{n=-\infty}^{\infty} \tilde{\xi}'_{oj} \tilde{G}'_{22}(\gamma, n) \\ \times \tilde{E}_z(n) = 0, \quad j=1,2,\dots,n_z. \end{aligned} \quad (\text{A18d})$$

Treat the even-mode coupled slot situation by placing (A10) into (A18) grouping spectrally summed terms using  $\tilde{\eta}'_{ej} = \sin(\alpha_n s) \tilde{\eta}_{ej}$ ,  $\tilde{\eta}'_{oj} = \cos(\alpha_n s) \tilde{\eta}_{oj}$ ,  $\tilde{\xi}'_{ej} = \cos(\alpha_n s) \tilde{\xi}_{ej}$ ,  $\tilde{\xi}'_{oj} = \sin(\alpha_n s) \tilde{\xi}_{oj}$ , and defining

$$X_{11}^{jm11} = \sum_{n=-\infty}^{\infty} \sin^2(\alpha_n s) \tilde{\eta}_{ej} \tilde{G}'_{11} \tilde{\eta}_{em} \quad (\text{A19a})$$

$$X_{11}^{jm12} = \sum_{n=-\infty}^{\infty} \sin(\alpha_n s) \cos(\alpha_n s) \tilde{\eta}_{ej} \tilde{G}'_{11} \tilde{\eta}_{om} \quad (\text{A19b})$$

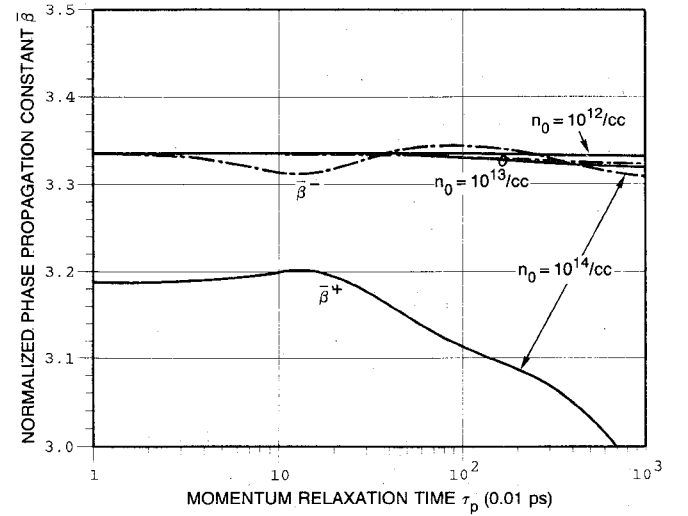


Fig. 14. Variation of  $\bar{\beta}^+$  and  $\bar{\beta}^-$  against  $\tau_p$  for the Fig. 13(a) case.  $n_0 = 10^{12}, 10^{13}$ , and  $10^{14}/\text{cc}$ .

$$X_{11}^{jm21} = \sum_{n=-\infty}^{\infty} \cos(\alpha_n s) \sin(\alpha_n s) \tilde{\eta}_{oj} \tilde{G}'_{11} \tilde{\eta}_{em} \quad (\text{A19c})$$

$$X_{11}^{jm22} = \sum_{n=-\infty}^{\infty} \cos^2(\alpha_n s) \tilde{\eta}_{oj} \tilde{G}'_{11} \tilde{\eta}_{om} \quad (\text{A19d})$$

$$X_{12}^{jm11} = \sum_{n=-\infty}^{\infty} \sin(\alpha_n s) \cos(\alpha_n s) \tilde{\eta}_{ej} \tilde{G}'_{12} \tilde{\xi}_{em} \quad (\text{A19e})$$

$$X_{12}^{jm12} = \sum_{n=-\infty}^{\infty} \sin^2(\alpha_n s) \tilde{\eta}_{ej} \tilde{G}'_{12} \tilde{\xi}_{om} \quad (\text{A19f})$$

$$X_{12}^{jm21} = \sum_{n=-\infty}^{\infty} \cos^2(\alpha_n s) \tilde{\eta}_{oj} \tilde{G}'_{12} \tilde{\xi}_{em} \quad (\text{A19g})$$

$$X_{12}^{jm22} = \sum_{n=-\infty}^{\infty} \cos(\alpha_n s) \sin(\alpha_n s) \tilde{\eta}_{oj} \tilde{G}'_{12} \tilde{\xi}_{om} \quad (\text{A19h})$$

$$X_{21}^{jm11} = \sum_{n=-\infty}^{\infty} \cos(\alpha_n s) \sin(\alpha_n s) \tilde{\xi}_{ej} \tilde{G}'_{21} \tilde{\eta}_{em} \quad (\text{A19i})$$

$$X_{21}^{jm12} = \sum_{n=-\infty}^{\infty} \cos^2(\alpha_n s) \tilde{\xi}_{ej} \tilde{G}'_{21} \tilde{\eta}_{om} \quad (\text{A19j})$$

$$X_{21}^{jm21} = \sum_{n=-\infty}^{\infty} \sin^2(\alpha_n s) \tilde{\xi}_{oj} \tilde{G}'_{21} \tilde{\eta}_{em} \quad (\text{A19k})$$

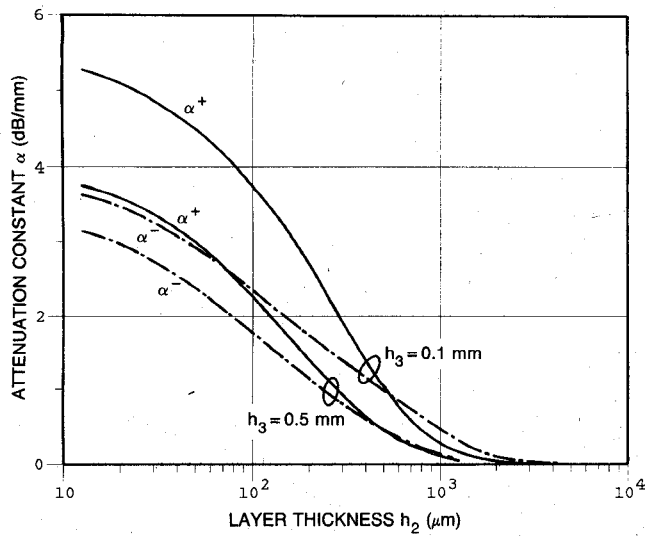
$$X_{21}^{jm22} = \sum_{n=-\infty}^{\infty} \sin(\alpha_n s) \cos(\alpha_n s) \tilde{\xi}_{oj} \tilde{G}'_{21} \tilde{\eta}_{om} \quad (\text{A19l})$$

$$X_{22}^{jm11} = \sum_{n=-\infty}^{\infty} \cos^2(\alpha_n s) \tilde{\xi}_{ej} \tilde{G}'_{22} \tilde{\xi}_{em} \quad (\text{A19m})$$

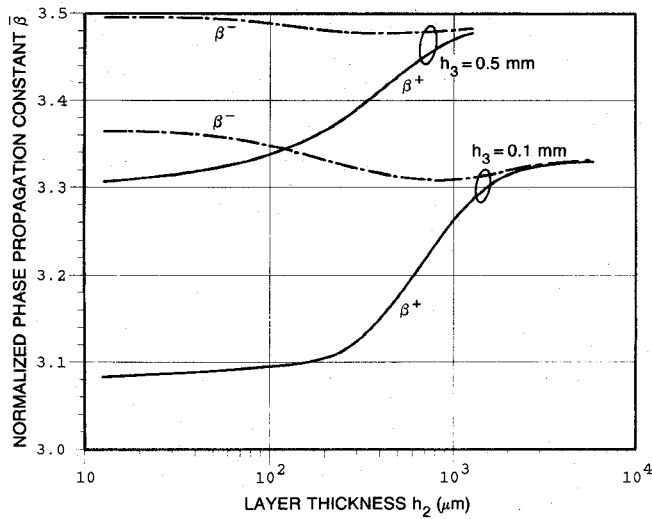
$$X_{22}^{jm12} = \sum_{n=-\infty}^{\infty} \cos(\alpha_n s) \sin(\alpha_n s) \tilde{\xi}_{ej} \tilde{G}'_{22} \tilde{\xi}_{om} \quad (\text{A19n})$$

$$X_{22}^{jm21} = \sum_{n=-\infty}^{\infty} \sin(\alpha_n s) \cos(\alpha_n s) \tilde{\xi}_{oj} \tilde{G}'_{22} \tilde{\xi}_{em} \quad (\text{A19o})$$

$$X_{22}^{jm22} = \sum_{n=-\infty}^{\infty} \sin^2(\alpha_n s) \tilde{\xi}_{oj} \tilde{G}'_{22} \tilde{\xi}_{om} \quad (\text{A19p})$$



(a)



(b)

Fig. 15. (a) Variation of  $\alpha^+$  and  $\alpha^-$  against  $h_2$  for the Fig. 12(a) case. Here  $B_0 = 10$  kG and  $w = 0.2$  mm with  $h_3 = 0.1$  or  $0.5$  mm. (b) Variation of  $\beta^+$  and  $\beta^-$  against  $h_2$  for the Fig. 15(a) case.

one obtains

$$\sum_{m=1}^{n_x} S_{11}^{jm} \begin{bmatrix} a'_{m1} \\ b'_{m1} \end{bmatrix} + \sum_{m=1}^{n_z} S_{12}^{jm} \begin{bmatrix} c'_{m1} \\ d'_{m1} \end{bmatrix} = 0, \quad j = 1, 2, \dots, n_x \quad (\text{A20a})$$

$$\sum_{m=1}^{n_x} S_{21}^{jm} \begin{bmatrix} a'_{m1} \\ b'_{m1} \end{bmatrix} + \sum_{m=1}^{n_z} S_{22}^{jm} \begin{bmatrix} c'_{m1} \\ d'_{m1} \end{bmatrix} = 0, \quad j = 1, 2, \dots, n_z \quad (\text{A20b})$$

where  $S_{11}^{jm}$ ,  $S_{12}^{jm}$ ,  $S_{21}^{jm}$ , and  $S_{22}^{jm}$  are  $n_x \times n_x$ ,  $n_x \times n_z$ ,  $n_z \times n_x$ , and  $n_z \times n_z$  sized matrices, and

$$S_{kl}^{jm} = \begin{bmatrix} X_{kl}^{jm11} & X_{kl}^{jm12} \\ X_{kl}^{jm21} & X_{kl}^{jm22} \end{bmatrix}. \quad (\text{A21})$$

Consequently, the anisotropic determinantal equation can

be expressed in compact form as

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} v_1 v_2 \cdots v_{n_x} w_1 w_2 \cdots w_{n_z} \end{bmatrix}^T = 0 \quad (\text{A22})$$

where

$$v_m = \begin{bmatrix} a'_{m1} \\ b'_{m1} \end{bmatrix}, \quad w_m = \begin{bmatrix} c'_{m1} \\ d'_{m1} \end{bmatrix} \quad (\text{A23})$$

or, in the most abbreviated form, as

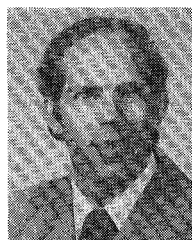
$$S \begin{bmatrix} v \\ w \end{bmatrix} = 0. \quad (\text{A24})$$

It is the solution to  $\det S(\gamma) = 0$  which we seek,  $\gamma = \gamma^+, \gamma^-$ . For the odd-mode coupled slot situation, use of (A13) leads to the (A24) form if all  $\sin(\alpha_n s)$  and  $\cos(\alpha_n s)$  factors are interchanged in (A19) and double primed notation is adopted in (A23).

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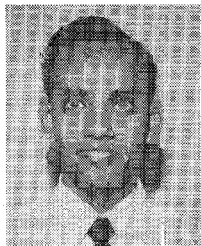


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